IMPROVING THE MATHEMATICS ACHIEVEMENT OF ALL STUDENTS

What We Know from the Research
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WHAT'S AT STAKE

The No Child Left Behind Act (NCLB), signed into law by President George W. Bush on January 8, 2002, will affect all aspects of teaching and learning mathematics. This education reform act includes four major provisions: accountability, flexibility, options for parents, and research-based curriculum and teaching methods.

Student progress and achievement will be measured according to state tests in mathematics and reading that will be given to every child, every year in grades 3–8. The data will be available in annual report cards on school performance. These report cards will give parents information about the quality of their children’s schools, the qualifications of teachers, and their children’s progress in key subjects. Progress toward meeting annual state learning targets will be in place to ensure that all groups of students reach proficiency within twelve years.

School districts and schools that fail to make adequate yearly progress (AYP) targets will, over time, be subject to improvement, corrective action, and restructuring measures aimed at getting them back on course to meet state standards. The stakes have never been higher for individual schools and classroom teachers. Never has it been more important that research-based practices are implemented in classrooms and that professional development of teachers is properly focused.

With NCLB as a backdrop, this issue brief looks at how research has informed our understanding of effective mathematics teaching and learning, and how we can use this knowledge to ensure that all students become proficient in this key subject. In particular, the brief will focus on the areas of 1) the mathematics curriculum, 2) balanced instruction, 3) the language of mathematics, 4) effective instructional practices, 5) assessment, and 6) promoting teacher excellence.

By the end of the 2005-06 school year, all states must ensure that every core subject classroom teacher is “highly qualified.”

(U.S. Department of Education, Office of the Secretary, No Child Left Behind, 2001.)

1. Curriculum

An effective mathematics curriculum expects more from students, and is balanced with respect to mathematical strands and conceptual understanding, problem solving, and computational fluency.

Higher Expectations for Students

An effective mathematics curriculum expects more of students than has historically been expected of them in traditional school mathematics. "Research indicates that students can learn more mathematics than is usually offered them in the early grades, so the U.S. elementary school mathematics curriculum could be made more challenging" (National Research Council 2001).
At first, these new rigorous standards for students might seem unattainable. However, the research indicates that "older views that young children are incapable of complex reasoning have been replaced by evidence that children are capable of sophisticated levels of thinking and reasoning..." (Bransford et al. 2000). In light of this research, effective mathematics curricula, for example, place a strong emphasis on algebraic thinking beginning in kindergarten. The research also indicates "that when students believed to be less than capable academically are given an opportunity to learn, they can in fact do so" (Knapp, Shields, and Turnbull 1995).

Today the math wars continue, with some arguing that mathematics programs should focus on traditional skills and numbers, while others argue that traditional skills are no longer relevant and that other mathematics should be emphasized. Neither side is correct. As the diagram from the National Council of Teachers of Mathematics Professional Standards indicates, the mathematics curriculum is about more than number, but number remains a significant component of a balanced curriculum, particularly in the elementary grades (NCTM 2000). A highly effective curriculum is also balanced with respect to conceptual understanding, computational fluency, problem solving and reasoning, and communication.

Do students still need to learn how to compute with paper and pencil now that calculators and computers are available?

YES. The widespread availability of calculators has greatly reduced the need for performing complex calculations with paper and pencil. But students need to understand what is happening in these complex calculations. They need to perform simpler computations with paper and pencil because that helps them develop math proficiency.

Conceptual Understanding
"Understanding refers to a student's grasp of fundamental mathematical ideas. Students with understanding know why a mathematical idea is important and when it is useful to use that idea..." (National Research Council 2001). Skills without conceptual understanding are meaningless and learning with understanding is more efficient. As the research indicates, "when students learn the concepts that underlie solutions not only is there greater retention, but also an increased likelihood that ideas will be used in new situations" (Cawelti 1999).

Problem Solving and Reasoning
Without the ability to solve problems, computational skill and conceptual understanding have no utility. Thus, problem solving is an integral part of all mathematics learning. Instructional programs should enable all students...
to "apply and adapt a variety of appropriate strategies to solve problems" (NCTM 2000).

Typical problem-solving strategies include using diagrams, working a simple problem, looking for patterns, working backward, making a list, and guessing and checking. However, the key is that these and other strategies "must receive instructional attention if students are expected to learn them" (NCTM, 2000).

Research indicates that instruction devoted to these strategies can make a difference in student achievement. "Students whose teachers emphasize higher-order thinking skills, particularly those that involve the development of strategies to solve different types of problems, outperform their peers by about 40 percent of a grade level" (Wenglinsky 2000).

Mastery of Skills

Conceptual understanding without computational fluency is inefficient—in fact, the two are interconnected. Procedural fluency and conceptual understanding "are interwoven. Understanding makes learning skills easier. By the same token, a certain level of skill is required to learn many mathematical concepts with understanding..." (National Research Council 2001).

Understanding, problem solving, and computational skills are three equal components of an effective mathematics program, and the research indicates that students immersed in a curriculum focused on conceptual understanding, as well as procedural skills, perform above average on state mathematics assessments (Knapp, Shields, and Turnbull 1995).

### WHICH SIDE OF THE "MATH WARS" IS CORRECT?

Reform efforts during the 1980’s and 1990’s downplayed computational skill, emphasizing instead that students should understand and be able to use math. In extreme cases, students were expected to invent math with little or no assistance. Reactions to these efforts led to increased attention to memorization and computational skill, with students expected to internalize procedures presented by teachers or textbooks. The clash of these contrasting positions has been called the "math wars."

Which position is correct? Neither. Both are too narrow. When people advocate only one strand of proficiency (conceptual or procedural), they lose sight of the overall goal. Such a narrow treatment of math may well be one reason for the poor performance of U.S. students in national and international assessments.

Math instruction cannot be effective if it is based on extreme positions. Students become more proficient when they understand the underlying concepts of math, and they understand the concepts more easily if they are skilled at computational procedures.

U.S. students need more skill and more understanding along with the ability to apply concepts to solve problems, to reason logically, and to see math as sensible, useful, and doable. Anything less leads to knowledge that is fragile, disconnected, and weak.


### MATH VERSUS ENGLISH VOCABULARY

The word “difference” could confuse a young student:

**Question**
What is the difference between 4 and 3?

**Student Answer**
Four is even and three is odd.

The correct response is 1.


### 3. THE LANGUAGE OF MATHEMATICS

Without communication students have no way to share, discuss, and refine ideas; and teachers have fewer opportunities to assess student understanding. The National Council of Teachers of Mathematics (NCTM) Professional Standards for School Mathematics also recommends that “instructional programs should enable all students to communicate their mathematical thinking coherently to others” (NCTM 2000).
Communication in mathematics should focus on learning the language of mathematics and on writing and discussing mathematics to explain and justify procedures and solutions, and to demonstrate understanding.

Reading

There is little doubt that students who have strong language skills also perform better in mathematics. Researchers (Montis 2000) have found that language processes play a vital role in the development of concept flexibility necessary for success in mathematics.

"Educators have tended to underestimate the contribution that reading can make to mathematics instruction...In fact, the explicit belief that the 'reading' component creates an obstacle to learning mathematics may have contributed to the trend toward minimizing the use of reading in mathematics instruction" (Borasi and Siegel 2000).

Reading has traditionally been downplayed in mathematics because reading mathematics is particularly challenging. It is challenging because mathematics has its own unique vocabulary and because it involves reading symbols in addition to letters.

Math Vocabulary

National recommendations call for instructional programs that "enable all students to use the language of mathematics to express their mathematical ideas precisely" (NCTM 2000).

Reading mathematics is particularly challenging for many students because "many mathematical words are unusual, some terms have different meanings in everyday usage than in technical mathematics, and some are used in more than one way. However, terms, phrases, and symbols are essential in communicating mathematical ideas...[and] vital for children's mathematical learning" (Rubenstein and Thompson 2002).

Reading Symbols

"It is often said that mathematics is a symbolic language...the symbols of mathematics, like the letters or characters of other languages, the written language of mathematics" (Usiskin/NCTM 1996).

CONFUSING VOCABULARY

<table>
<thead>
<tr>
<th>Metric Unit</th>
<th>Non-Metric Unit</th>
</tr>
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<tbody>
<tr>
<td>Meter</td>
<td>Decimeter</td>
</tr>
<tr>
<td>Millimeter</td>
<td>Kilometer</td>
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<tr>
<td>Centimeter</td>
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And then comes the Geometry Unit:

- Perimeter
- How many in a peri?

Mathematics requires students to be proficient at decoding not only words, but also numeric and nonnumeric symbols. Readers must shift from sounding out words such as plus or minus to recognizing their symbolic counterparts, + and - (Barton and Heidema 2002).

"Teachers and students often use informal terminology in place of more formal terms used in the text. Consequently, students do not gain familiarity with the technical vocabulary that they will be expected to know. For instance, if the teacher continually substitutes the phrase 'number on top' for the term numerator in an attempt to make the meaning clearer for students in the short run, he may be doing them a disservice in the long run. Students need to learn the technical terms they will encounter in future course work" (Barton and Heidema 2002).

Formal vocabulary and mathematically correct definitions must be taught and are emphasized in an effective curriculum. Research indicates that knowledge of mathematics vocabulary affects achievement in mathematics, particularly in the area of problem solving (Helwig, Rozek-Tedesco, Tindale, Heath, and Almond 1999).

NUMERIC SYMBOLS

These fractions are equivalent because the quotient of each fraction is the same:

\[
\begin{align*}
\frac{-10}{5} &= -2 \\
\frac{10}{-5} &= -2 \\
\frac{-1(10)}{5} &= -2
\end{align*}
\]

NONNUMERIC SYMBOLS

Sometimes a symbol represents more than one word:

- \(\text{subtract, take away, minus, negative, difference}\)
- \(\text{is less than or equal to}\)
- \(\text{ plus or minus}\)
- \(\text{is perpendicular to}\)
Writing as Communication

Writing in mathematics is an effective instructional tool, particularly if it focuses on student reasoning. "One of the best ways for students to improve their reasoning skills is to explain or justify their solutions to others. Once a procedure is developed, students should sometimes be asked to explain and justify that procedure" (National Research Council 2001).

Some examples of effective writing prompts in mathematics include:
Write a rule for _____. Explain why your rule works.
How are x and y similar? How are they different?
Explain why _____ is correct. (or incorrect)
Write about a situation where you might need an estimate instead of an exact answer.

Discussion as Communication

Discussion within lessons is crucial. It is here [during discussion] that teachers will decide what aspects of children's thinking to bring forward, relate, and extend. It is also in this setting that students will have an opportunity to reflect on their own thinking and consider it in light of other students' thinking (Empson/NCTM 2002).

Questioning

Effective teachers ask a lot of questions and involve students in class discussions. They also do the following:

• plan questions when preparing lessons;
• focus questions on searching for student understanding; and
• ask questions for different purposes—clarifying, redirecting, summarizing, extension, and reflection.
(Sutton and Kruger 2002)

Mathematical Conversations

Mathematical conversations model a way of thinking and learning mathematics. They also offer a unique way to get students to talk about mathematical ideas and concepts. Some ideas may not be entirely correct or not expressed correctly. However, research from cross-national studies suggests that classroom discussion of incorrect ideas is valuable to student learning (Ma 2003). Communication is a hallmark of classrooms in which students build mathematical understandings (Hiebert 1997).

4. Effective Instructional Practices

Lesson Structure

Much debate centers on forms and approaches to teaching: "direct instruction" versus "inquiry," "teacher centered" versus "student centered," "traditional" versus "reform." However, the effectiveness of mathematics teaching and learning does not rest in simple labels. Effective teaching—teaching that fosters the development of mathematical proficiency over time—can take a variety of forms (National Research Council 2001).

Teacher Effectiveness

Teacher effectiveness is 10 to 20 times more important in terms of student learning than factors such as class size, per-pupil spending, ethnic makeup, and poverty (Sanders and Horn 2000). Teachers have an incredible impact on student achievement. Nearly 50 percent of student achievement is based on the cumulative effect of teacher sequence. Sanders (2001) has shown that if students have a "bad" teacher, four years later they suffer from residual effects no matter how effective the subsequent teachers. If a student has a "bad" math teacher two years in a row, it is nearly impossible for the student to recover.

The Bookends of Effective Instruction

One way that teachers can help students identify the key mathematical point of a lesson is to describe the goal of the lesson. A second way to help students recognize key ideas in a lesson is a summary statement at the end of a lesson. On average, mathematics teachers in Japan summarized a higher percentage of problems per lesson than in any other country.

Lesson Effectiveness

Dixon et al. (1998) note the following as key elements of an effective lesson:

- **Central Focus**—the lesson's specific mathematical content and goal is clear.
- **Phase One**—the teacher engages students by demonstrating, explaining, questioning, conducting a discussion, etc.
- **Phase Two**—guided practice, monitoring and adjustment.
- **Phase Three**—independent practice.
- **Closure**—ties the mathematical results of the activities to the central goal of the lesson.

"Assessment should support the learning of important mathematics and furnish useful information to both teachers and students" (NCTM 2000).

What Do We Know from Research?

- We know that at the classroom level, frequent assessment is useful, particularly when teachers are given help on what they should do for children who aren’t performing well (Whitehurst 2003).
- We know that "assessments must be followed by high-quality, corrective instruction designed to remedy whatever learning errors the assessment identified. To charge ahead knowing that students have not learned certain concepts or skills well would be foolish" (Guskey 2003).
- We know that diagnostic assessment tied to intervention is essential because high performing schools tend to "implement comprehensive systems to monitor individual student progress and provide extra support to students as soon as it’s needed" (Education Trust 1999).
- We also know that diagnostic assessment tied to intervention supports NCTM’s Equity Principle. "Equity does not mean that every student should receive identical instruction; instead it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM 2000).

Appropriate Use of Manipulatives

Mathematics educators have been encouraging teachers to allow students to use manipulatives for decades (Adding It Up, 2001 p.45). However, the value of using manipulative materials to investigate a concept depends not only on whether manipulatives are used, but also on how they are used with students (Cawelti 1999).

Teachers need to make explicit connections between concrete experiences, concepts, and symbols. "If students do not see the connections among object, symbol, language, and idea, using manipulatives becomes just one more thing to learn rather than a process leading to a larger mathematical learning goal" (National Research Council 2001).

"Students whose teachers conduct hands-on learning activities [purposefully with the intent of illustrating concepts] outperform their peers by more than 70 percent of a grade level in mathematics" (Wenglinsky 2000).

A PROFOUND UNDERSTANDING OF FUNDAMENTAL MATHEMATICS

Limited subject matter knowledge restricts a teacher's capacity to promote conceptual learning among students. Even a strong belief of “teaching mathematics for understanding” cannot remedy or supplement a teacher's disadvantage in subject matter knowledge.


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How Can Content Knowledge Be Improved?

The latest research confirms the profound importance of subject-matter focus in designing high-quality professional development (Garet, Porter, Desimone, Birman and Yoon 2001).

"We want teachers to have a firm grasp of the following characteristics of mathematics, namely:

(1) that precise definitions form the basis of any mathematical explanation, and without explanations mathematics becomes difficult to learn;

(2) that logical reasoning is the lifeblood of mathematics, and one must always ask why as well as find out the answer; and

(3) that concepts and facts in mathematics are tightly organized as part of a coherent whole so that the understanding of any fact or concept requires also the understanding of its interconnections with other facts and concepts" (Wu 2001).

Liping Ma offers the following recommendations for achieving reform:

• Address Teacher Knowledge and Student Learning at the Same Time. Chinese teachers develop a profound understanding of mathematics (PUFM) when they teach school mathematics. It would be unrealistic to expect U.S. teachers' subject matter knowledge of school mathematics to be improved before mathematics education in school is improved.

• Enhance the Interaction Between Teachers' Study of School Mathematics and How To Teach It. U.S. teachers have less working time outside the classroom than Chinese teachers. They do not have enough time and support to think through thoroughly what they are to teach and how to teach it thoughtfully.

• Refocus Teacher Preparation. What we should do is rebuild substantial school mathematics with a more comprehensive understanding of the relationship of fundamental mathematics and new advanced branches of the discipline.

• Understand the Role of Textbooks. Both textbooks and state frameworks and standards are subject to different interpretations. Teacher materials can explain the way topics were selected and sequenced. They can also provide very specific information about the nature of students' responses to particular activities. Teachers can both use and go beyond the textbook.

• Understand the Key to Reform. The real mathematical thinking that goes on in a classroom depends heavily on the teacher's understanding of mathematics. If a teacher's own knowledge of the mathematics taught in elementary school is limited to procedures, how can we expect his or her classroom to have a tradition of inquiry mathematics?

The change that we are expecting can occur only if we work on changing teachers' knowledge of mathematics (Ma 1999).

Requirements of No Child Left Behind, particularly adequate yearly progress targets, have significantly raised the accountability stakes for schools and school districts. Never has it been more important that educators avoid extreme, trendy, and unproven educational fads. It is possible to raise the achievement of all students and meet the goals of No Child Left Behind by expecting more of all students, by implementing a balanced curriculum, and by focusing teacher professional development not only on effective instructional practices, but on teacher content knowledge as well.

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